

Worksheet for 2020-09-14

Problem 1. Letting a be a fixed constant, consider the function

$$f(x, y) = \begin{cases} \frac{(x+y)^a}{x^2+y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

Is f continuous if $a = 0$? If $a = 1$? If $a = 2$? If $a = 3$?

Problem 2. Consider $x^2 + y^2 + z^2 = 1$. Note that $\frac{\partial x}{\partial y}$ for example means to view the equation as implicitly defining x as a function of y and z , and then to take the partial derivative of that function with respect to y (i.e. treating z as constant).

Compute the quantities $\frac{\partial x}{\partial y}$, $\frac{\partial y}{\partial z}$, $\frac{\partial z}{\partial x}$, and then compute their product (notice that the answer is *not* 1, giving you an example for why you should not think of these expressions as “fractions”).